

Possible Color Octet Quark-Anti-Quark Condensate in the Instanton Model

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Abstract

Inspired by a recent proposal for a Higgs description of QCD we study the possible formation of a color-octet/ flavor-octet quark-anti-quark condensate in the instanton liquid model. For this purpose we calculate two-point correlation functions of color-singlet and octet quark-anti-quark operators. We find long range order in the standard $\langle \bar{\psi}\psi \rangle$ channel, but not in the color-octet channel. We emphasize that similar calculations in lattice QCD can check whether or not a color-flavor locked Higgs phase is realized in QCD at zero temperature and baryon density.

It was recently argued that confinement in QCD can be understood in terms of an effective Higgs description where $SU(3)$ gauge invariance is spontaneously broken by a color-octet quark-anti-quark condensate [1–4] (see [5,6] for earlier speculations in this direction)

$$\langle \bar{\psi}(\lambda^a)_C (\lambda^a)_F^T \psi \rangle = 2 \left(\delta_i^a \delta_j^b - \frac{1}{3} \delta^{ab} \delta_{ij} \right) \langle \bar{\psi}_i^a \psi_j^b \rangle. \quad (1)$$

Here, $(\lambda^a)_C$ is a color $SU(3)$ matrix and $(\lambda^a)_F$ is a flavor $SU(3)$ matrix. The condensate (1) breaks the local gauge symmetry as well as the $SU(3)_L \times SU(3)_R$ chiral symmetry but

leaves a diagonal $SU(3)$ unbroken. This symmetry acts as the physical flavor symmetry. The Higgs field (1) allows for a remarkably simple description of the gross features of the QCD spectrum. Gluons acquire a mass via the Higgs mechanism and carry the flavor quantum numbers and charges of the vector meson octet. The quark degrees of freedom transform as an octet and a heavier singlet and carry the quantum numbers of the baryon octet and singlet.

This type of Higgs description was originally proposed as a complementary picture of QCD at large baryon chemical potential [7,8]. In that case, the primary Higgs field is given by the color-flavor locked diquark condensate

$$\langle \psi_i^a C \gamma_5 \psi_j^b \rangle = \phi \left(\delta_i^a \delta_j^b - \delta_i^b \delta_j^a \right). \quad (2)$$

This condensate has the same residual color-flavor symmetry as (1) but also breaks the $U(1)$ of baryon number. At large baryon density this is not a concern and simply corresponds to baryon superfluidity. The diquark condensate is dynamically generated by attractive interactions in the color anti-triplet quark-quark channel. At very large chemical potential this attraction is generated by one-gluon exchange [9] while at intermediate density instanton induced interactions are likely to play a role [10,11]. Because the color-octet quark-anti-quark condensate (1) is consistent with the symmetries of the color-flavor locked diquark phase we expect this operator to have a non-zero expectation value in the high density phase [12–14]. This expectation is borne out by explicit calculations [12,13] but the value of the color-octet condensate is strongly suppressed with respect to the primary Higgs field, $\langle \bar{\psi}(\lambda^a)_C(\lambda^a)_F^T \psi \rangle \simeq \langle \bar{\psi} \psi \rangle \ll \langle \psi C \gamma_5 (\lambda^a)_C(\lambda^a)_F \psi \rangle$. The suppression is due to an approximate Z_2 symmetry in the high density phase.

At zero baryon density the dynamical origin of a possible color-octet quark-anti-quark is unclear. Both one-gluon exchange and instantons are repulsive in this channel. Nevertheless, given the attractiveness of the scenario outlined in [1–4] it seems worthwhile to investigate this question beyond perturbation theory. In this note, we report on a calculation using the instanton liquid model [15,16]. This model correctly accounts for $\langle \bar{\psi} \psi \rangle$ at zero baryon

density and the formation of a diquark condensate at high density. The color-octet quark-anti-quark condensate at high density is induced by instantons. Also, Wetterich suggested that an instability in the instanton induced effective potential in three flavor QCD might be the dynamical origin of the color-octet quark-anti-quark condensate [3].

First we have to discuss how to identify the Higgs phase characterized by (1). The color-octet condensate (1) is not a gauge invariant operator and therefore it cannot be used as an order parameter. The simplest alternative is the square of the octet condensate. This is similar to studying the vev of the square of the $SU(2)$ Higgs field in the standard model. In the present case, however, this is not very useful. The square of the octet condensate receives large contributions from fluctuations associated with ordinary chiral symmetry breaking. This is suggested by the factorization approximation [17] which gives

$$\langle (\bar{\psi}(\lambda^a)_C(\lambda^a)_F^T \psi)^2 \rangle \simeq \frac{8}{81} \langle \bar{\psi} \psi \rangle^2. \quad (3)$$

In the instanton model $\langle (\bar{\psi}(\lambda^a)_C(\lambda^a)_F^T \psi)^2 \rangle$ is about 2-3 times larger than this estimate, but even larger deviations from factorization are observed in other Lorentz-scalar four-quark operators. Instead of the square of the octet condensate we propose to study the color-octet quark-anti-quark correlation function. If QCD can be described in terms of the Higgs picture advocated in [1] we expect to observe long range order in this correlation function. We should note that the color-octet correlation function is also not a gauge invariant object. This problem can be addressed by calculating the correlator in some fixed gauge or by inserting two gauge strings. One might worry that if the gauge strings are included the correlator will no longer approach a constant even if there is long range order. Nevertheless, if the Higgs description makes sense there has to be a clear difference between the color-octet correlator and a generic gauge invariant correlation function without long range order.

We have calculated the correlation functions in the color-singlet/ flavor-singlet, color-singlet/ flavor-octet and color-octet/ flavor-octet channel. The correlation functions are defined by

$$\Pi_{[1][1]}(x, y) = -\langle \text{Tr} [S^{ab}(x, y) S^{ba}(y, x)] \rangle + \langle \text{Tr} [S^{aa}(x, x)] \text{Tr} [S^{bb}(y, y)] \rangle, \quad (4)$$

$$\Pi_{[1][8]}(x, y) = -\langle \text{Tr} [S^{ab}(x, y) S^{ba}(y, x)] \rangle, \quad (5)$$

$$\Pi_{[8][8]}(x, y) = -4\langle \text{Tr} [S^{aa}(x, y) S^{bb}(y, x)] \rangle + \frac{4}{3}\langle \text{Tr} [S^{ab}(x, y) S^{ba}(y, x)] \rangle, \quad (6)$$

where $S^{ab}(x, y)$ is the quark propagator with color indices a, b and the traces are taken over Dirac indices. We have assumed exact flavor symmetry. The flavor-singlet correlation function $\Pi_{[1][1]}$ is strongly attractive and expected to approach $\langle \bar{\psi}\psi \rangle^2$ as $x-y$ tends to infinity. The flavor-octet correlator $\Pi_{[1][8]}$ is expected to decay exponentially with a characteristic mass $m_{[1][8]} \simeq m_{a_0} \simeq 1$ GeV. Depending on whether the Higgs description is valid we expect the color-flavor octet correlator $\Pi_{[8][8]}$ to behave similar to $\Pi_{[1][1]}$ or to $\Pi_{[1][8]}$.

Figures 1 and 2 show the three correlation functions (4-6) measured in quenched and unquenched instanton liquid simulation. In both cases we observe long range order in the $\bar{\psi}\psi$ channel, $\Pi_{[1][1]}(x) \rightarrow \text{const.}$ From the asymptotic value of the correlation function we find $\langle \bar{\psi}\psi \rangle \simeq -(250 \text{ MeV})^3$ in the quenched case and $-(220 \text{ MeV})^3$ in the unquenched case. The flavor-octet $\Pi_{[1][8]}$ correlation function decays exponentially. In the quenched calculation the correlator becomes unphysical for $x > 0.5$ fm. In the unquenched calculation the screening mass is consistent with 1 GeV.

We observe that the color-flavor octet correlator $\Pi_{[8][8]}$ also decays exponentially. There is no sign of long range order. In the unquenched calculation the screening mass is 700 MeV, while it is even larger in the quenched calculation. We have also calculated the correlator with the gauge links included. We observe no qualitative difference. In fact, the screening mass is slightly increased.

In summary we observe no evidence for long range order in the color-flavor octet channel. We suggest that lattice calculations of the correlation function (4-6) can check whether the Higgs picture suggested in [1] is realized in nature.

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FIGURES

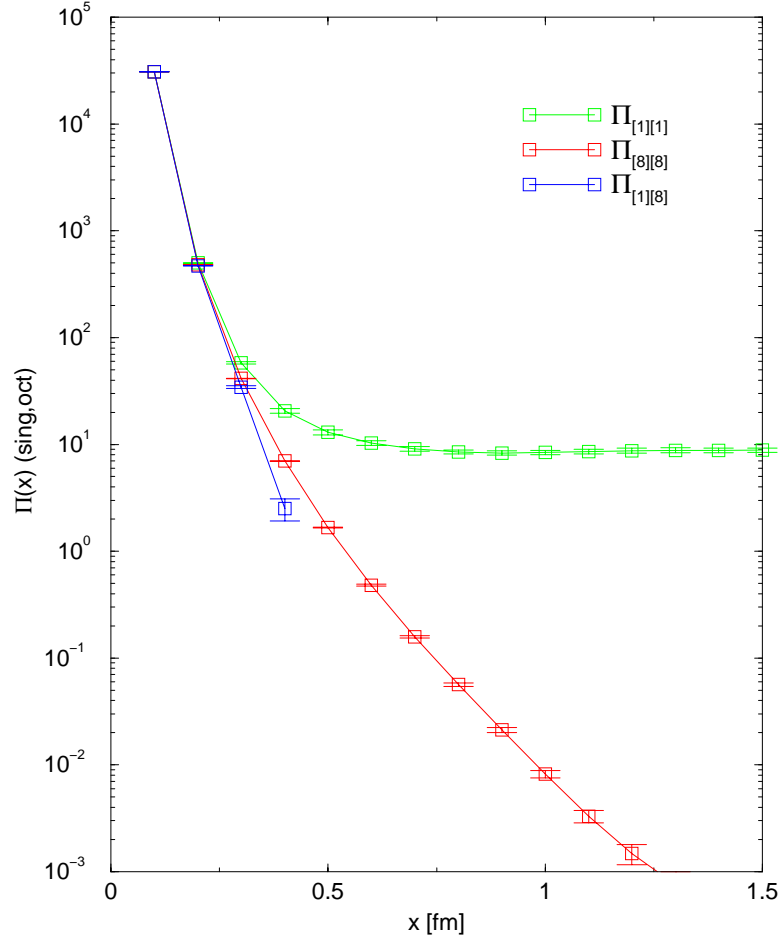


FIG. 1. Correlation functions in the color-singlet/ﬂavor-singlet ($[1][1]$), color-octet/ﬂavor-octet ($[8][8]$) and color-singlet/ﬂavor-octet ($[1][8]$) channel. The correlators were calculated in a quenched instanton ensemble. The function $\Pi_{[8][8]}$ was multiplied by a factor $3/32$ in order to normalize the short distance behavior of all correlation functions in the same way.

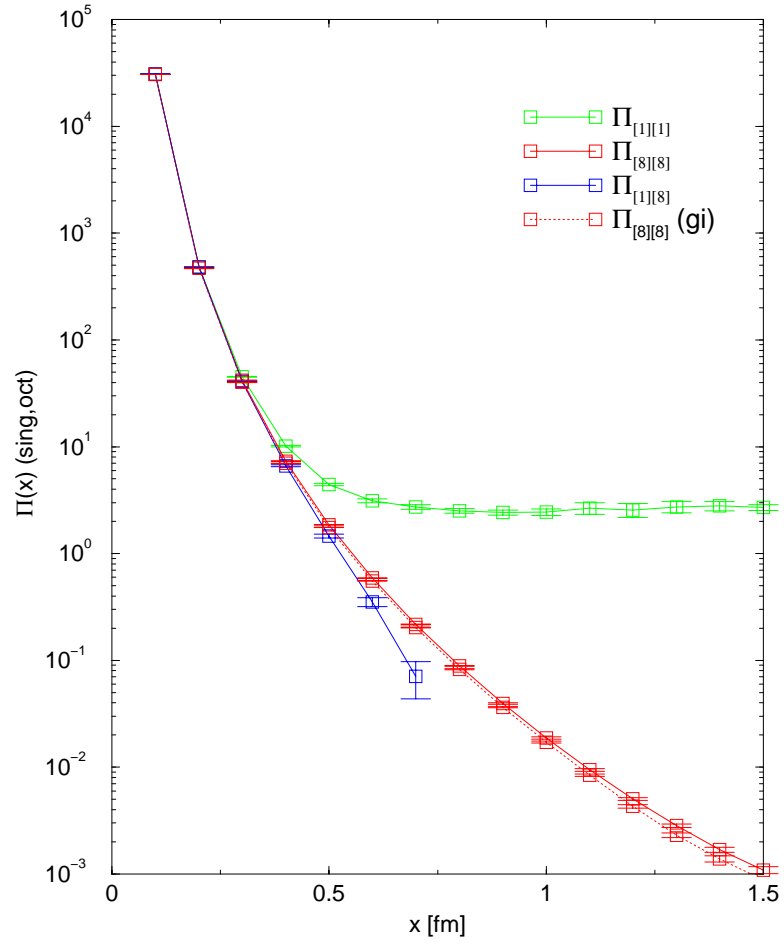


FIG. 2. Same correlators as in Figure 1 calculated in an unquenched instanton ensemble.

$\Pi_{[8][8]}(\text{gi})$ denotes the color-octet correlator with the gauge links included.